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BFS-based Symmetry Breaking Predicates for DFA Identification



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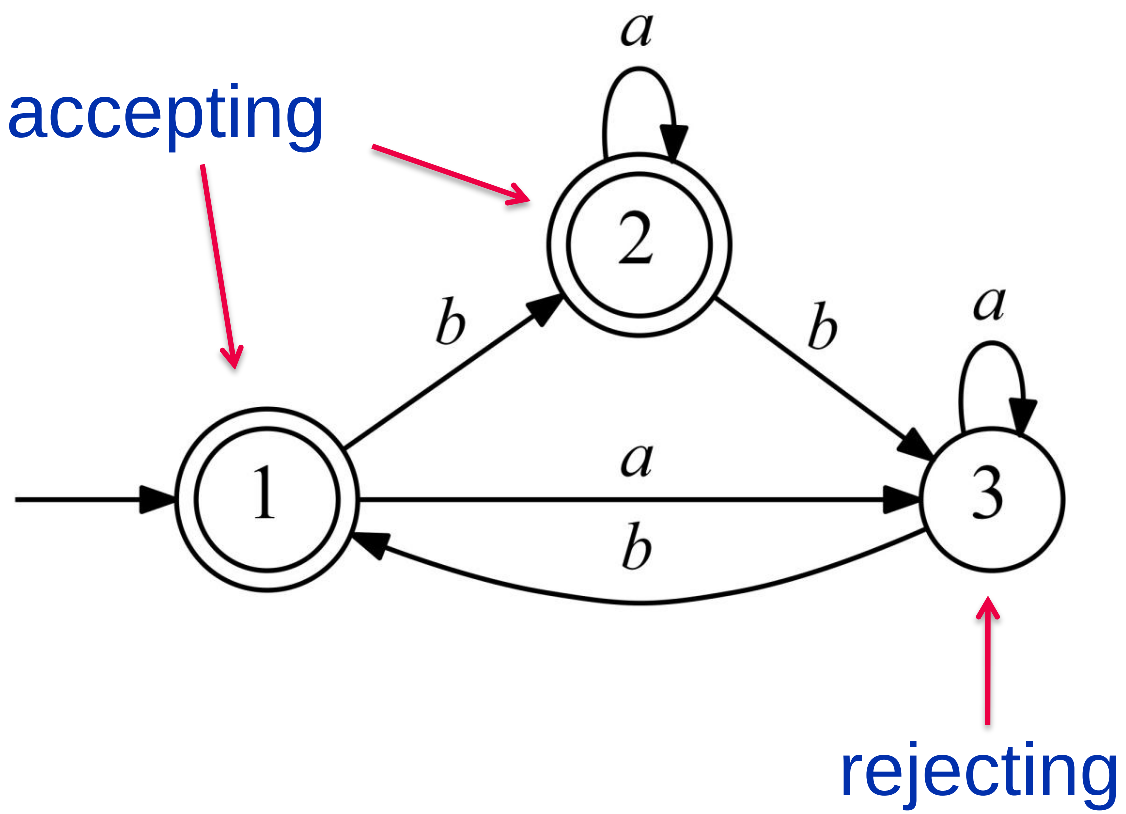
Outline

- Introduction
- DFASAT algorithm overview
- Handling noise in DFASAT
- BFS-based symmetry breaking for DFASAT
- Experiments
- Conclusions

Deterministic Finite Automata (DFA)

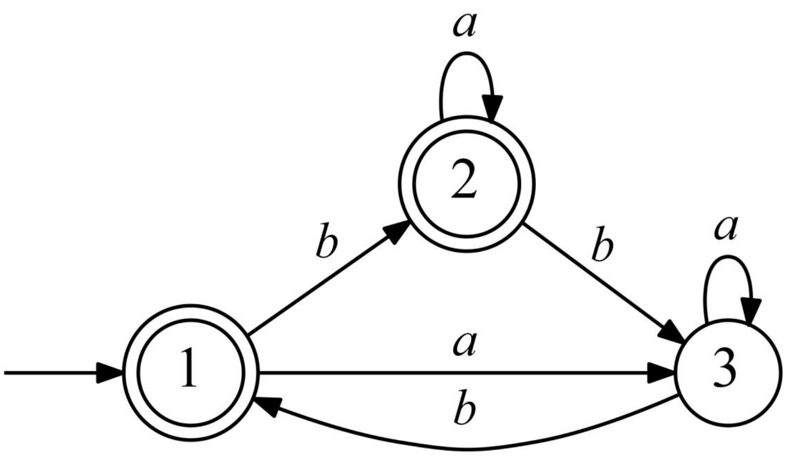


- S_+
  - ab
  - b
  - ba
  - bbb
- S_-
  - abbb
  - baba

DFA Identification Problem

S_+={ab, b, ba, bbb}

S_-={abbb, baba}



- Identifying a **minimal** DFA is NP-hard [Gold, 1978]

DFA Identification From Noisy Data

- K string labels are randomly flipped

S_+={ab, b, ba, bbb}; S_-={abbb, baba}

S_+={ab, b, ba}; S_-={abbb, baba, bbb}

Previous Research

- Evolutionary algorithm with smart state labeling [Lucas et al., 2005]
  - State of the art for noisy case
- **DFASAT [Heule & Verwer, 2010]**
  - **State of the art for noiseless case**

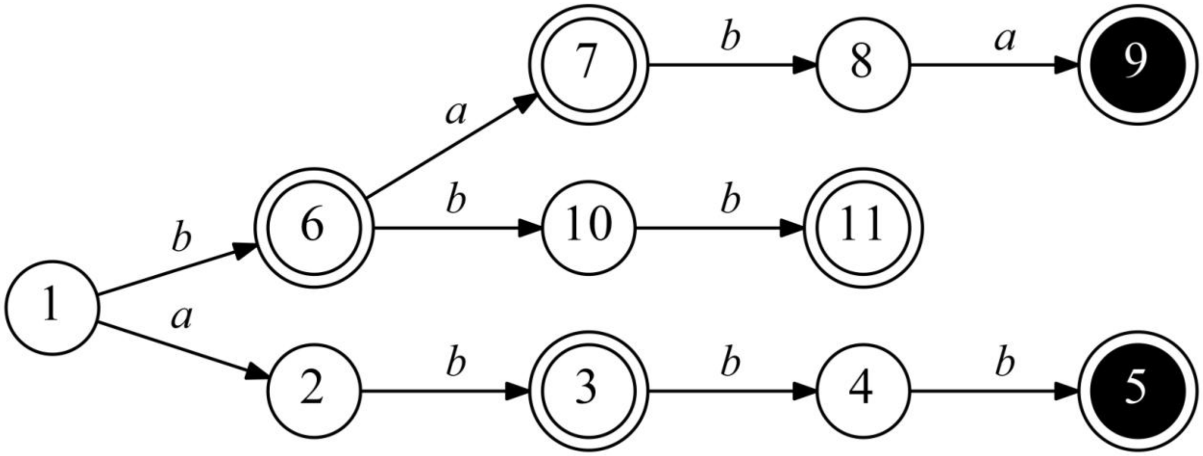
Our contribution

- We focus on DFASAT
- Augment DFASAT to handle **noisy data**
- Augment DFASAT with **new symmetry breaking predicates**

DFASAT [Heule & Verwer, 2010]

1. Augmented Prefix Tree Acceptor construction
2. Consistency Graph construction
3. CNF Boolean Formula construction
4. SAT-solver execution
5. DFA reconstruction from satisfying assignment

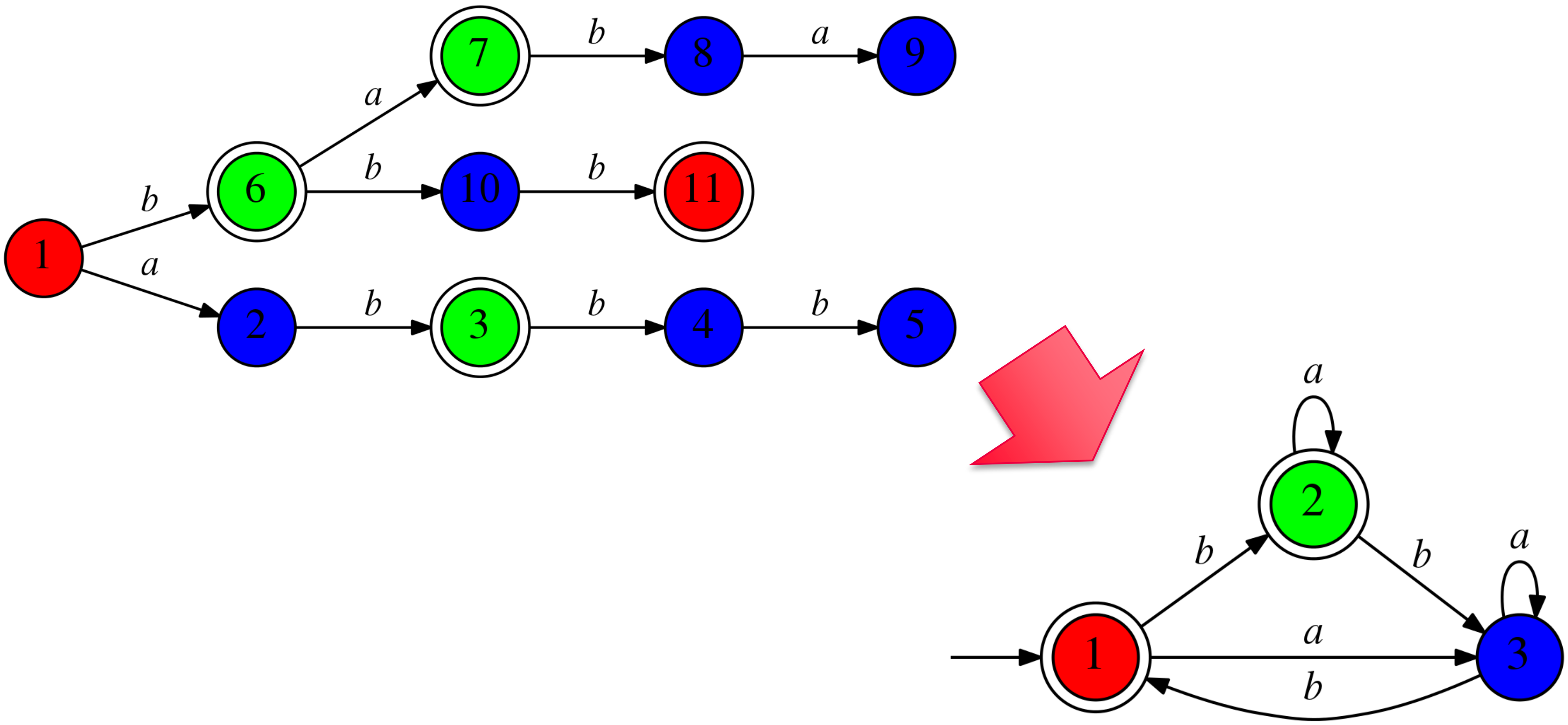
Augmented Prefix Tree Acceptor



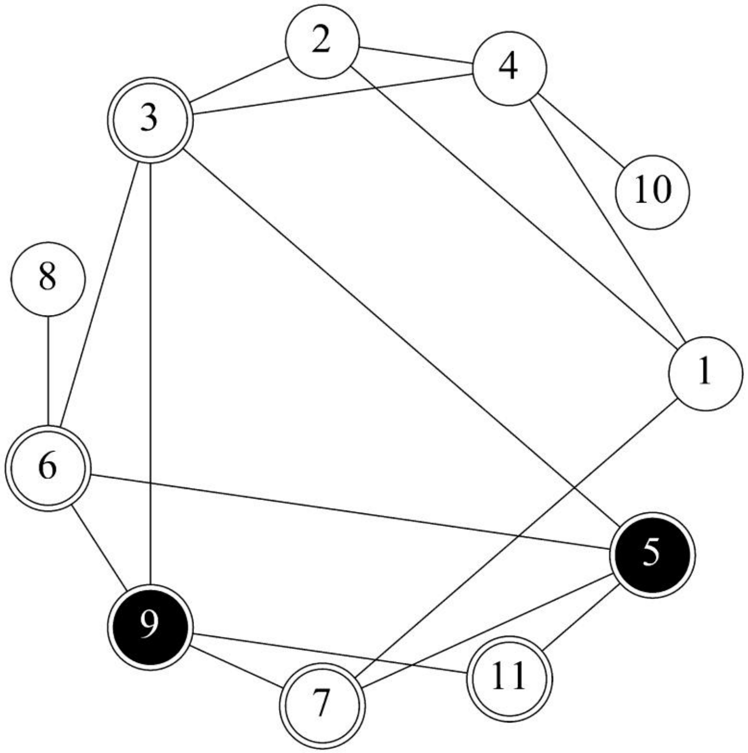
S_+
- ab
- b
- ba
- bbb

S_-
- abbb
- baba

Main idea: APTA coloring



Consistency Graph



- Nodes – same as APTA states
- Two nodes are connected if they cannot be merged into one DFA state
- Only exists in the noiseless case

Variables

- **Color** variables $x_{v,i} \equiv 1$ iff APTA state v has color i
- **Parent relation** variables $y_{l,i,j} \equiv 1$ iff DFA transition with symbol l from state i ends in state j
- **Accepting color** variables $z_i \equiv 1$ iff DFA state i is accepting

Types of clauses (1)

V_+ – accepting states
V_- – rejecting states

- **Accepting** states colors $x_{v,i} \Rightarrow z_i, \; v \in V_+$
- **Rejecting** states colors $x_{v,i} \Rightarrow \neg z_i, \; v \in V_-$
- Each state has **at least** one color $x_{v,1} \vee x_{v,2} \vee \ldots \vee x_{v,C}$
- Each state has **at most** one color $\neg x_{v,i} \vee \neg x_{v,j}, \; i < j$

Types of clauses (2)

p(*v*) – parent of APTA state v
l(*v*) – incoming symbol of APTA state *v*

- A DFA transition is set when a state and its parent are colored

$$x_{p(v),i} \wedge x_{v,j} \Rightarrow y_{l(v),i,j}$$

- Each DFA transition must target **at least** one state

$$y_{l,i,1} \vee y_{l,i,2} \vee \dots \vee y_{l,i,C}$$

- Each DFA transition can target **at most** one state

$$y_{l,i,j} \Rightarrow \neg y_{l,i,k}, \; j < k$$

Types of clauses (3)

- State color is set when DFA transition and parent color are set

$$y_{l(v),i,j} \wedge x_{p(v),i} \Rightarrow x_{v,j}$$

- Colors of two states connected with an edge in the consistency graph must be different

$$x_{v,i} \Rightarrow \neg x_{w,i}, (v,w) \in E$$

Noisy DFA Identification

- K random attribution labels are *flipped*

S_+={ab, b, ba, bbb}; S_-={abbb, baba}

S_+={ab, b, ba}; S_-={abbb, baba, bbb}

Noisy DFA Identification: Issues

- Consistency graph is undefined
- We do not know the exact labels of strings

- How can we modify the described translation to deal with noise?

Noisy DFA Identification (2)

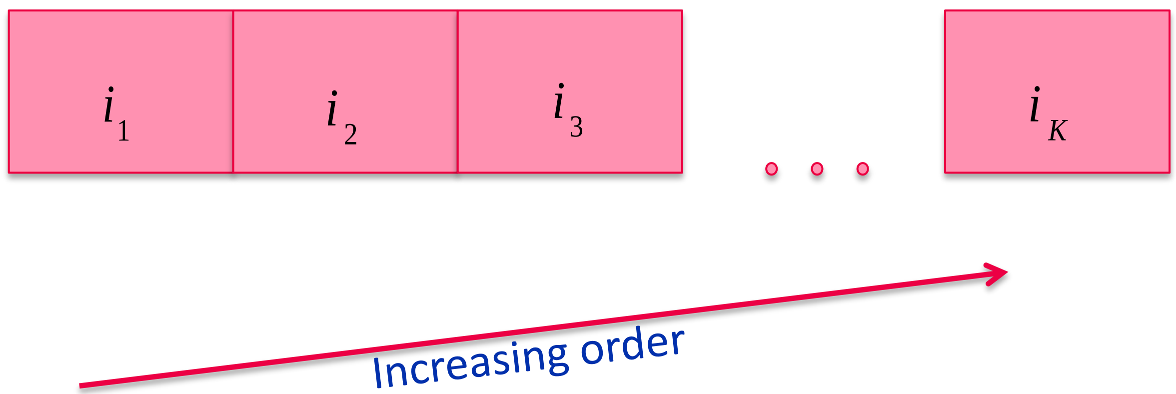
- New variables f_v
- $f_v \equiv 1$ iff the label of state v can (but <u>does not have to</u>) be incorrect (flipped)
- Modify clauses for state colors

$$x_{v,i} \Rightarrow z_i,\ v \in V_+ \qquad \longrightarrow \qquad \neg f_v \Rightarrow (x_{v,i} \Rightarrow z_i),\ v \in V_+$$

$$x_{v,i} \Rightarrow \neg z_i,\ v \in V_- \qquad \longrightarrow \qquad \neg f_v \Rightarrow (x_{v,i} \Rightarrow \neg z_i),\ v \in V_-$$

Noisy DFA Identification (3)

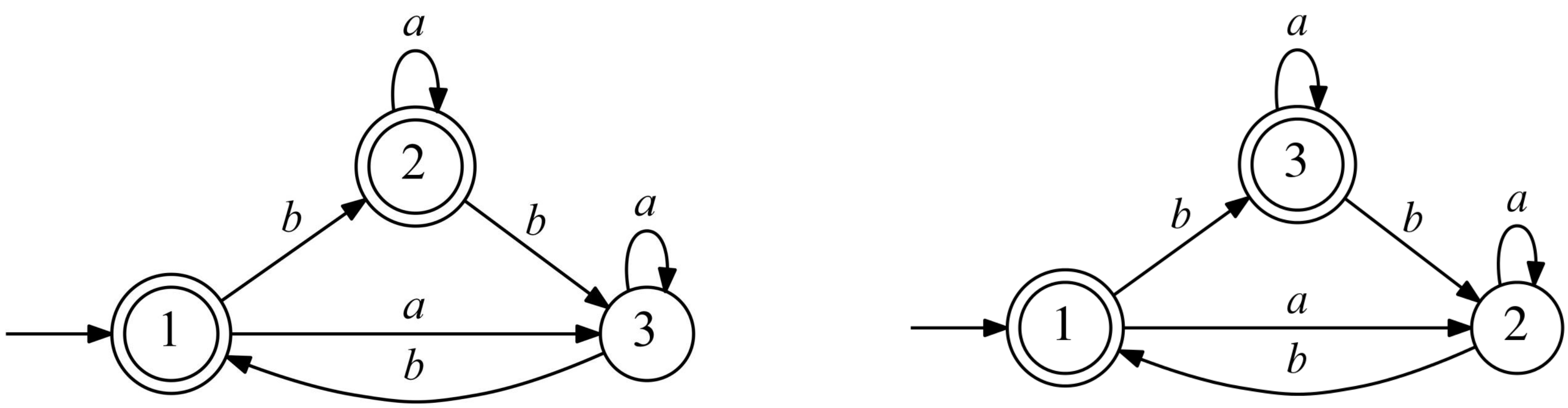
- Array of length *K*
- Numbers of APTA states for which that can be flipped



- Some extra variables and clauses for representing that as a Boolean formula; *order encoding* method used

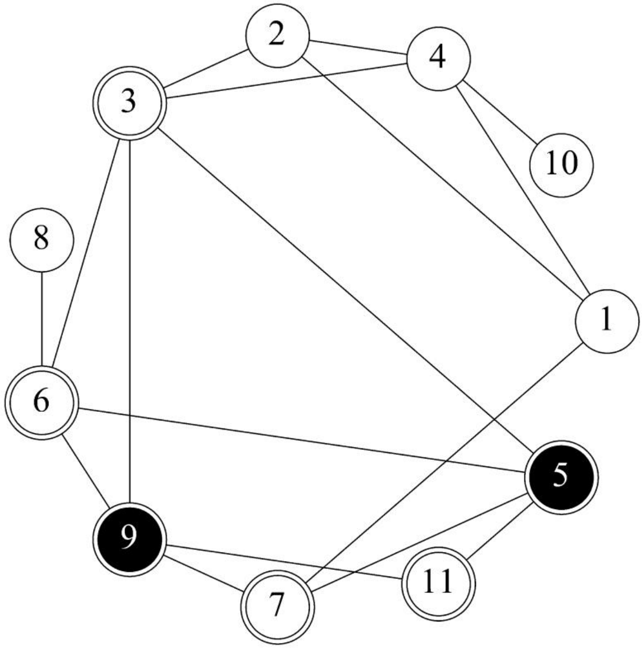
Symmetry breaking

- Many optimization problems exhibit symmetries
- Here: groups of isomorphic DFA

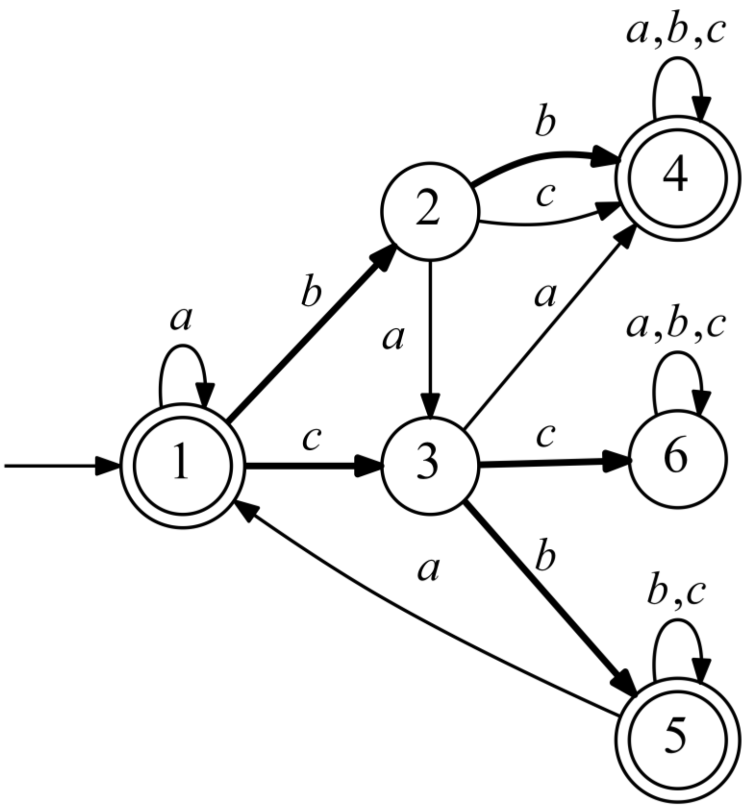


Max-clique symmetry breaking [Heule & Verwer, 2010]

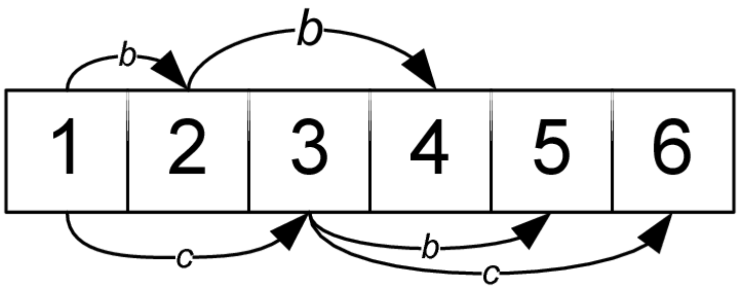
- Find a big clique in the CG with fast heuristic algorithm
- Fix colors of clique states in the APTA
- Note: not applicable in the noisy case



BFS-based Symmetry Breaking Predicates



BFS-enumerated DFA



BFS queue

BFS-based Symmetry Breaking Predicates

- Idea – force the DFA to be BFS-enumerated
- Already used in several algorithms

- How do we encode BFS-enumeration in SAT?

Additional variables

- **Parents** variables $p_{j,i} \equiv 1$ iff state i is the parent of state j in the BFS-tree

- **Transition** variables $t_{j,i} \equiv 1$ iff there is a transition between states i and j

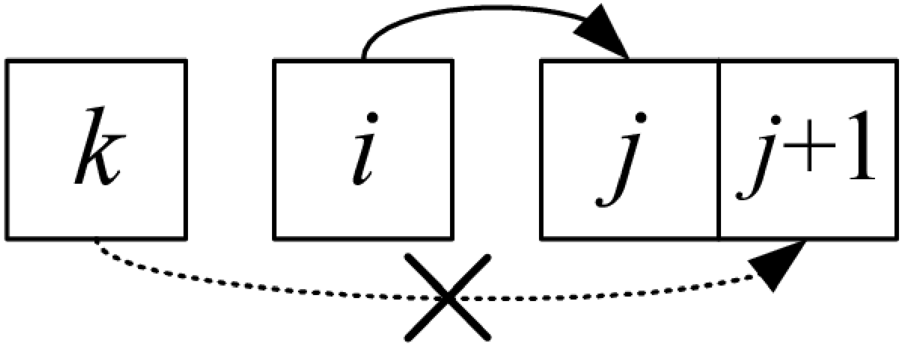
Ordering parents

- Each state except initial one must have a **parent** with a smaller number

$$p_{j,1} \vee p_{j,2} \vee \ldots \vee p_{j,j-1},\ 2 \le j \le C$$

- In BFS-enumeration states' **parents** must be ordered

$$p_{j,i} \Rightarrow \neg p_{j+1,k},\ 1 \le k < i < j < C$$



Ordering children

- Transition variables: there is a transition between states *i* and *j*

$$t_{i,j} \Leftrightarrow y_{l_1,i,j} \vee \ldots \vee y_{l_L,i,j},\ i < j$$

- State *j* was enqueued while processing the state with minimal number *i* among states that have a **transition** to *j*

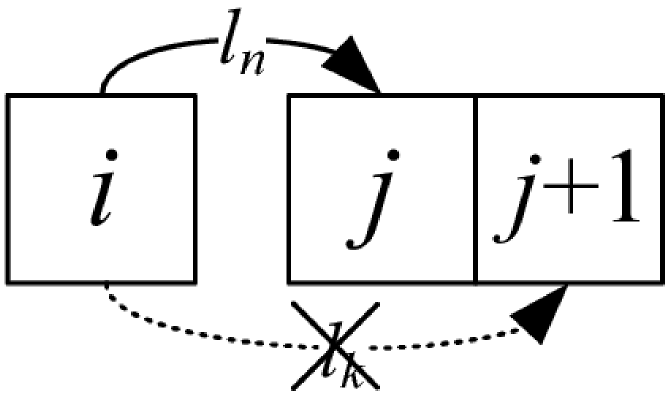
$$p_{j,i} \Leftrightarrow (t_{i,j} \wedge \neg t_{i-1,j} \wedge \ldots \wedge \neg t_{1,j}),\ i < j$$

Ordering transitions

- Minimal symbol variables

$$m_{l_n,i,j} \Leftrightarrow y_{l_n,i,j} \wedge \neg y_{l_{n-1},i,j} \wedge \ldots \wedge \neg y_{l_1,i,j},\ i < j$$

- Arranging consecutive states *j* and *j+1* with the same **parent** *i* in the alphabetical order of **minimal symbols** on **transitions** between them and *i*



$$p_{j,i} \wedge p_{j+1,i} \wedge m_{l_n,i,j} \Rightarrow \neg m_{l_k,i,j+1},\ i < j,\ k < n$$

Experimental setup

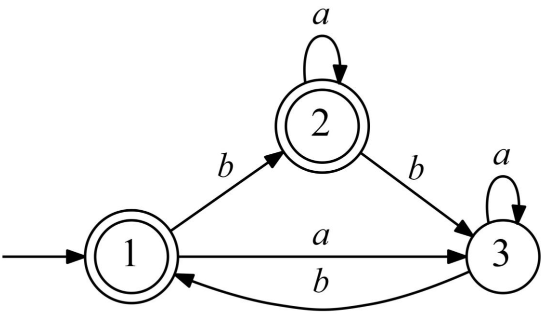
- Random data sets
- Binary alphabet
- *TL* – time limit (*TL* = 1800 seconds)
- *lingeling* SAT-solver
- Mean time among 100 launches of experiments

Noiseless DFA Identification

- DFASAT with max-clique symmetry breaking clearly outperforms our method

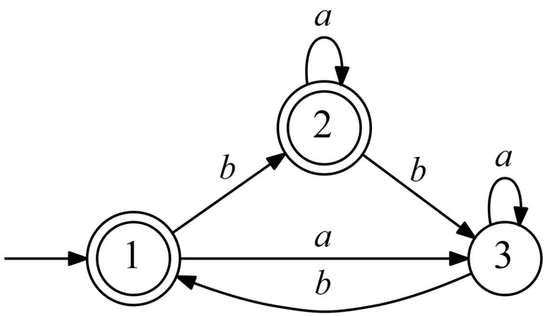
Noisy DFA Identification when target DFA exists

- *N* – size of the DFA used for generating input set of strings
- *N* – size of the target DFA



N states

S_+={ab, b, ba, bbb}
S_-={abbb, baba}



N states

Noisy DFA Identification, $S = 10N$ strings

Number of states	Noise level, %	BFS, s	DFASAT, s	EA, s
5	2	**0.22**	0.38	1.22
5	4	**0.59**	0.9	1.1
6	2	**1.05**	2.44	2.94
6	4	3.34	7.82	**2.85**
7	1	**4.34**	10.83	21.36
7	3	**17.22**	143.66	19.16
8	1	**17.89**	31.58	30.29
8	2	163.92	225.31	**19.8**

Noisy DFA Identification, *S* = 25*N* strings

Number of states	Noise level, %	BFS, s	DFASAT, s	EA, s
5	1	**0.54**	0.64	2.77
5	2	2.42	4.33	**1.80**
6	1	**6.3**	11.95	11.65
6	2	13.3	43.54	**4.8**
7	1	31.01	114.95	**17.24**
7	2	286.76	TL	**13.11**
8	1	239.46	404.32	**21.73**

Noisy DFA Identification, *S* = 50*N* strings

Number of states	Noise level, %	BFS, s	DFASAT, sec	EA, s
5	1	**4.2**	7.59	6.07
5	2	12.87	22.36	**3.05**
6	1	20.76	52.5	**20.39**
6	2	107.94	309.22	**11.28**

Noisy DFA identification when the target DFA does not exist

- (N + 1) – size of the DFA used for generating input set of strings
- N – size of the target DFA

- Note: the state-of-the-art EA cannot determine that a DFA consistent with a given set of strings **does not exist**

Noisy DFA identification when the target DFA does not exist, $S = 50N$ strings

N	K	BFS, s	DFASAT, s	Passed BFS, %	Passed DFASAT, %
5	1	**11.57**	257.13	100	100
5	2	**46.42**	1296.71	100	30
6	1	**110.05**	TL	100	0
6	2	**581.73**	TL	100	0
7	1	**995.27**	TL	89	0
7	2	TL	TL	0	0

Conclusion

- Exact solution for noisy DFA identification
- New symmetry breaking predicates based on BFS
  - Applicable in the noisy case
  - Greatly speed up the discovery of non-existence of a DFA
- Implementation
  - http://github.com/ctlab/DFA-Inductor

Acknowledgements

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Thank you for your attention!

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