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SYSTEM OF TRANSFORMATIONS OF CERTAIN REPRESENTATIONS OF BOOLEAN FUNCTIONS

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Three forms of representation of Boolean functions are considered: the nonrepetitive Boolean formula, the linear binary graph, and the orthogonal disjunctive normal form. A one-to-one correspondence among them is exhibited, and a complete system of equivalent transformations is studied. The role of these transformations in computer-aided programming of logical devices is noted.

The following problems arise in the automation of the programming of logical devices [1]: to estimate the complexity of a binary graph (BG) of a given Boolean formula (BF) in the basis AND, OR, NOT; to translate a BF into a BG; to enumerate the paths in a given BG (testing of a BG); to retranslate a BF of a given BG; to retranslate a BF of a given enumeration of the paths of a BG; to form a BG from a given enumeration of its paths.

Because an arbitrary BF can always be reduced by a change of variables to a nonrepetitive Boolean formula (NBF) represented in the same basis and containing the same number of letters [2], the BG corresponding to it will be linear (LBG) [3-5]. Therefore the above problems involving NBFs and LBGs can be solved, and it is possible to use orthogonal disjunctive normal forms (ODNF) to enumerate the paths of a LBG [6].

Thus to solve the above problems it is necessary to consider a complete system of one-to-one equivalent transformations of the three forms of representation of Boolean functions: NBF, LBG, and ODNF (Fig. 1).

Methods are known for carrying out some of these transformations; for example, a transformation of a NBF into a LBG is studied in [3-5], and a transformation of a NBF into a ODNF in [6]. Transformations of LBGs into NBFs and ODNFs, and transformations of ODNFs into NBFs and LBGs are not considered in practice. Therefore one can say that a complete system of transformations has never been studied from a single point of view.

It is the purpose of this article to remedy this deficiency.

We enumerate the basic peculiarities of a LBG. Its conditional vertices correspond, one-to-one, to the letters of the right-hand side of the NBF and are placed linearly in the same order as the letters in the NBF. A LBG, besides h conditional vertices, also contains two operator vertices, where h is the number of letters in the right-hand side of the NBF. The conditional vertices of the LBG are connected by arcs which are always directed to the right. Adjacent conditional vertices are always connected by an arc. We will always give the operator vertex "y = 1" the number h + 1, and the vertex "y = 0" the number h + 2, where y is the notation for the NBF.

In what follows, we assume that the NBF is positively monotone, and that its structure is fixed.

1. TRANSFORMATION OF A NBF INTO A LBG

In [4, 5] an algorithm is given which is based on the replacement of fragments of a NBF by intermediate variables and the construction of a LBG by the composition of its fragments corresponding to these variables. Therefore, the number of steps in the transformation of the NBF into the LBG is not proportional to the number of letters in the NBF but depends in an essential way on the structure of the NBF. We propose a simple method for accomplishing this transformation with a linear bound for the complexity of the number of steps.

We construct the framework of the LBG from the linear arrangement of the vertices corresponding to the letters of the NBF in order of their appearance from left to right in its

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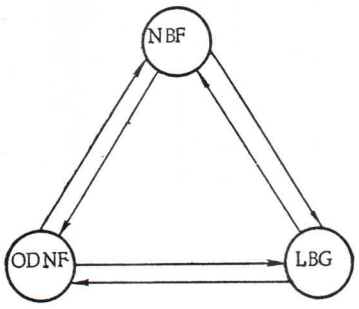

Fig. 1. Complete system of equivalent transformations of the three forms of representation of Boolean functions.

right-hand side. Adjacent conditional vertices are connected by arcs which are oriented to the right. We will call them basic arcs in what follows. We will call the last conditional vertex to the right the output. We will not consider operator vertices in what follows, but we will use instead the term "output" (unit or zero).

We mark off the basic arcs in the following way. We designate the arc as unit (zero) if there is a conjunction (disjunction) sign after the corresponding letter in the NBF.

Since two arcs must issue from each conditional vertex ("vertex" from now on), the second (additional) arc must connect the given vertex with some vertex of the framework whose symbol is as yet unknown. To determine the symbol of this vertex, we introduce the concept of the remainder of the formula (RF) with respect to a letter of the NBF.

The RF of some letter of a NBF is a conjunction (disjunction) transformed by neglecting that part of the NBF located to the left of this letter and by including unpaired parentheses in the remaining part of the NBF consisting of more than one letter. In addition, the conjunction (disjunction) can contain an arbitrary formula in parentheses as a factor (term). For example, for a NBF of the form $y = x_1(x_2 \vee x_3x_4)x_5 \vee x_6x_7$ we get as RFs of its letters: $x_1(x_2 \vee x_3x_4)x_5$; $x_2 \vee x_3x_4$; $x_3x_4x_5$; x_4x_5; $x_5 \vee x_6x_7$; x_6x_7.

The unknown vertex with which the one under consideration is connected by the new arc corresponds to the letter on the right of the RF just obtained. If there is no letter to the right of this RF, then the new arc is connected to a unit output vertex if the RF is a disjunction and to a zero output vertex if the RF is a conjunction.

In the case where the formula is not monotonic, the LBG is constructed disregarding inverse, and then arcs issuing from vertices corresponding to letters with inverses are labeled with the opposite symbols.

Example 1. We construct the LBG for a given NBF of the form $y = (x, \bar{x}_2 \vee x_3)\bar{x}_4$. First we construct the LBG for the NBF without the negations $y = (x_1x_2 \vee x_3)x_4$. We construct the framework and mark the vertices, the basic arcs, and the arcs of the output vertex (Fig. 2a). We list the RFs for y: x_1x_2; $x_2 \vee x_3$; x_3x_4. Hence it follows that vertex x_1 must be connected to x_3 (Fig. 2b), x_2 to x_4 (Fig. 2c), and x_3 to a zero output (Fig. 2d). The arcs issuing from x_2 and x_4 must be relabeled with the opposite symbols because they appear in y with inverses (Fig. 2e).

Thus the LBG of a given NBF can be constructed in h (for a positively monotone formula) or h + 1 (for a nonmonotonic formula) steps. For the nonmonotonic formula in this example, the number of steps in the transformation is five (Fig. 2a-e).

2. TRANSFORMATION OF A NBF INTO AN ODNF

Known methods of orthogonalization (obtaining an ODNF from a given BF) are oriented, as a rule, toward the reduction of the BF to a DNF, from which one ODNF is constructed ([6], for example). Two basic rules are used to do this.

We first introduce the following notation: D is a disjunction, D' is the ODNF of a disjunction, $\bar{D}'$ is the ODNF of a negation of a disjunction, K is a conjunction, K' is the ODNF of a conjunction, and $\bar{K}'$ is the ODNF of a negation of a conjunction.