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GENERATING BOOLEAN FUNCTIONS VIA ARITHMETIC POLYNOMIALS

V. L. Artyukhov, V. N. Kondrat'ev,
and A. A. Shalyto

We will expand the concept of an arithmetic polynomial by introducing the "absolute value" operation. We propose methods for constructing a polynomial for Boolean functions. The conditions of polynomial linearity are defined.

1. Introduction

The possibility of using nontraditional methods to generate Boolean functions when microprocessors and microcomputers are used in logic control systems. These are based on the extensive capabilities of the indicated hardware to do arithmetic. These methods are, e.g., spectral methods, methods for computing threshold functions, computing by means of arithmetic polynomials (AP).

The latter group of methods was examined in [1, 2]. This work will further the issues in generating Boolean functions (BF) and systems of Boolean functions by arithmetic polynomials.

2. Expanding the Concept of An "Arithmetic Polynomial"

It is a well-known fact that a Boolean equation based on AND, OR, and NOT operations can be generated via an AP by replacing the logical operations with arithmetic operations according to the rules:

$$x_1 \& x_2 = x_1 x_2; \quad x_1 \vee x_2 = x_1 + x_2 - x_1 x_2; \quad \bar{x} = 1 - x.$$

Notice too, that if the Boolean functions f_1 and f_2 are orthogonal ($f_1 \cdot f_2 = 0$)

$$f_1 \vee f_2 = f_1 + f_2 = f_1 \oplus f_2.$$

We will prove a theorem that frequently makes it possible to simplify an AP obtained by means of the indicated rules.

Theorem 1. Let x_1, $x_2 \in \{0, 1\}$, and let N and k be random integers. Then

$$x_i = (x_i)^N,$$

$$x_i + (2^N - 2) x_1 x_2 + x_2 = (x_1 + x_2)^N,$$

$$(x_1 - x_2)^{2k} = |x_1 - x_2|,$$

$$(x_1 - x_2)^{2k+1} = x_1 - x_2.$$

are valid.

Proof. The first relation is obvious. We will prove the others. To do this, we consider the binomial theorem:

$$(x_1 + x_2)^N = x_1^N + N x_1^{N-1} x_2 + \frac{N(N-1)}{1 \cdot 2} x_1^{N-2} x_2^2 + \ldots + x_2^N.$$

Starting from the fact that when raising the sum of the variables x_1 and x_2 to a power the sum of the coefficients on the binomial is equal to 2^N and that Eq. (2) is valid, we can rewrite Eq. (6) as:

$$(x_1 + x_2)^N = x_1 + (2^N - 2) x_1 x_2 + x_2.$$

When raising the difference between the variables x_1 and x_2 to the Nth power the sum of the coefficients on the binomial is zero, and therefore

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