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to have taken place reasonably soon thereafter.
GENERATING BOOLEAN FUNCTIONS VIA ARITHMETIC POLYNOMIALS

V. L. Artyukhov, V. N. Kondrat'ev,
and A. A. Shalyto

We will expand the concept of an arithmetic polynomial by introducing the absolute value operation. We propose methods for constructing a polynomial for Boolean functions. The conditions of polynomial linearity are defined.

1. Introduction

The possibility of using nontraditional methods to generate Boolean functions when microprocessors and microcomputers are used in logic control systems. These methods are based on the extensive capabilities of the indicated hardware to do arithmetic computing by means of arithmetic polynomials (AP).

The latter group of methods was examined in [1, 2]. This work will further the issues in generating Boolean functions (BF) and systems of Boolean functions (BF) and arithmetic polynomials.

2. Expanding the Concept of an "Arithmetic Polynomial"

It is a well-known fact that a Boolean equation based on AND, OR, and NOT operations can be generated via an AP by replacing the logical operations with arithmetic operations according to the rules:

\[ x_1 \& x_2 = x_1 x_2; \quad x_1 \lor x_2 = x_1 + x_2 - x_1 x_2; \quad \bar{x} = 1 - x. \]

Notice too, that if the Boolean functions \( f_1 \) and \( f_2 \) are orthogonal (\( f_1 \cdot f_2 = 0 \)),

\[ f_1 \lor f_2 = f_1 + f_2 = f_1 \oplus f_2. \]

We will prove a theorem that frequently makes it possible to simplify an AP obtained by means of the indicated rules.

**Theorem 1.** Let \( x_1, x_2 \in \{0,1\} \), and let \( N \) and \( k \) be random integers. Then

\[ x_i = (x_i)^N, \]

\[ x_i + (2^N - 2)x_i x_i = (x_i + x_2)^N, \]

\[ (x_i - x_1)^N = |x_i - x_1| \]

\[ (x_i - x_1)^{N+1} = x_i - x_2, \]

are valid.

**Proof.** The first relation is obvious. We will prove the others. To do this, we use the binomial theorem:

\[ (x_i + x_2)^N = x_i^N + N x_i^{N-1} x_2 + \ldots + x_2^N. \]

Starting from the fact that when raising the sum of the variables \( x_1 \) and \( x_2 \) to \( N \), the sum of the coefficients on the binomial is equal to \( 2^N \) and that Eq. (2) is valid, can rewrite Eq. (6) as:

\[ (x_i + x_2)^N = x_i + (2^N - 2)x_i x_2. \]

When raising the difference between the variables \( x_1 \) and \( x_2 \) to the \( N \)th power the sum of the coefficients on the binomial is zero, and therefore

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