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COMPUTER METHODS

Multifunctional Logic Modules Consisting of Elements with Bilateral Conductance

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Abstract—Multifunctional logic modules consisting of elements with bilateral conductance are proposed; when realizing Boolean formulas in the basis {&, ∨, !} consisting of at most six letters, these modules have no element redundancy. If the basis has more than six letters, then the redundancy does not exceed 33%.

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INTRODUCTION

There exist different types of multifunctional logic modules consisting of functional elements (elements with unilateral conductance) [1–5], methods for their design [3, 6], and methods for synthesizing logic circuits from these elements [7, 8]. However, in designing modules of this class, as well as in synthesizing circuits from them, a superposition is usually used. As a result, it seems that some inputs of functional elements are lost, since they are connected not to input variables, but to outputs of preceding elements.

Elements with bilateral conductance whose logic is made "in wires" are free from this disadvantage. As a result, if we have, e.g., two AND elements (each with two inputs) consisting of elements with unilateral conductance, then, superposing them, we obtain an AND element with three inputs, while, by integrating elements with bilateral conductance by wires, an AND element with four inputs is constructed. Elements with bilateral conductance become especially useful in synthesizing multi-output structures, in realizing systems of Boolean functions.

In control systems, among elements with bilateral conductance, contact elements are most common in use. The very large dimensions of relay-contact elements, the low speed, and the high power consumption render their use as a bilateral-conductance component type unsuitable. Nevertheless, methods for synthesizing logic structures from elements with bilateral conductance do not recede into the background, since they can be used not only in relay-contact circuit engineering.

The objective of this work is to describe multifunctional logic modules consisting of elements with bilateral conductance without element redundancy up to a certain number q of letters in the realized Boolean formula in the basis {&, ∨, !}. With these values of q, the multifunctionality of these modules is provided only due to redundancy in the external outputs. As in modules consisting of elements with unilateral conductance, for large numbers of letters, both kinds of redundancy (element and for external outputs) take place. The proposed modules are usually aligned at outputs due to their identification, while at inputs they are aligned as modules consisting of elements with unilateral conductance.

1. MODULES THAT ARE UNIVERSAL IN THE CLASS OF ARBITRARY BOOLEAN FORMULAS IN THE BASIS {&, ∨, !}

Assuming that direct and inverse input variables are equally accessible, the proposed modules designed, as an illustration, from single-contact relays of the same type have the homogeneous structure shown in Fig. 1. Here, normally open contact *1* and winding *2* form relay *3*.

The considered class of modules with $2(q + 1)$ external outputs was first described in [9]. Such modules consisting of q elements (for $q \leq 6$) are universal in the class of Boolean formulas in the basis {&, ∨, !} consisting of q letters under equal access of direct and inverse input variables.[1] With these values of q, the modules have a redundancy only in the external outputs.

The proposed modules are significantly simpler than modules consisting of elements with unilateral conductance for the same values of q. For example, at $q = 5$, module [10] has 14 external contacts (two contacts are for supplying the power) and consists of 79 insulated-gate field-effect transistors, which are integrated into functional elements, while the proposed

[1] The possibility of delivering direct and inverse values of input signals is meant (*Editor's note*).

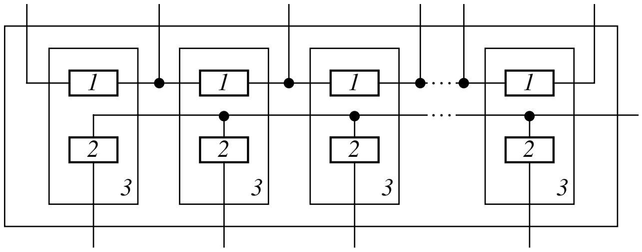

Fig. 1.

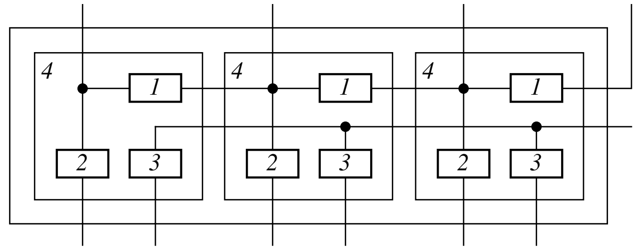

Fig. 2.

module for this value of q has 12 external contacts and includes only 5 elements. For $q = 7$, a module consisting of 7 elements realizes, by an alignment, 179 of 180 types of parallel–serial seven-contact circuits, whereas, to realize one circuit discussed below, a module consisting of 8 elements is needed. Hence, element redundancy appears in the proposed modules only for this value of q.

Note that a method for counting the number of parallel–serial circuits consisting of q contacts is suggested in [11], and their tabulation for $q \le 8$ was performed by the author of this paper and presented in [12]. The tabulation of the types of circuits made it possible to check the realizability of each circuit with the use of the proposed module. With an increase in q, the number of parallel–serial contact circuits that are realized by the proposed module consisting of q elements decreases. For example, at $q = 8$, by tuning a module consisting of 8 elements, we can realize 518 of feasible 522 circuits.

The reason why it is impossible to design modules that are irredundant in the number of elements for $q \ge 7$ is considered in the next section; there, it is also shown that, for an arbitrary value of q, a module of the proposed type consisting of elements whose number does not exceed q can be designed. If only direct input variables are accessible for a module, then, as a q-universal module for $q = 2$ or 3, a circuit of q switching contacts can be used [13]. In Fig. 2, such a module is shown as an example for $q = 3$; here, a switching contact, which is conditionally represented by normally open contact *1* and normally closed contact *2* and winding *3* form relay *4*. In [14], the alignment of this module to realize a representative for each of 20 P-types [13] of repetition-free Boolean formulas in the considered basis is described.

2. TOPOLOGICAL METHOD FOR REALIZING COMBINATIONAL CIRCUITS

Assume that direct and inverse input variables are equally accessible and that the modules whose structure is shown in Fig. 1 are given. We demonstrate that, using the property of bilateral conductance, it is possible to construct a method for synthesizing circuits from considered modules, which provides significantly smaller redundancy than that in the formula method [7, 13] for realizing Boolean formulas in the basis $\{\&, \vee, !\}$ including h letters, in modules consisting of elements with unilateral conductance that are universal in the class of formulas in the same basis including q letters ($q \le h$).

The proposed method based on the property of Eulerian graphs, which can be "passed" without detaching a pencil from paper [13], is called topological. A graph is called Eulerian if it has zero or two odd nodes [15, 16]. A node is called odd if an odd number of edges enter into it. From graph theory, it is known that, in the absence of odd nodes, a graph is "passable" if its passing begins at any even node, while, with two odd nodes, its passing must begin at one of these nodes. The well-known Hoang Tuy algorithm [15] generates an Eulerian chain in the case where the graph has two odd nodes. In an arbitrary graph, the number of odd nodes is even, and, if the graph has more than two such nodes, it is "impassable" [16].

These results of graph theory can be employed in realizing arbitrary multipole circuits that are constructed from normally open contacts with the use of the module shown in Fig. 1. If the realized circuit is Eulerian, then we must find an Eulerian path in it, which determines the alignment of the module, namely, the notation of contacts in the "chain" and a jumper between its external outputs. Since the access to any point in the chain is ensured, an arbitrary Eulerian circuit with q contacts can be irredundantly realized by such a chain with q contacts.

Figure 3 demonstrates the contact circuit, which realizes the system of Boolean formulas $z_1 = x_1 \vee x_2 \vee x_4$, $z_2 = x_3$. This circuit consisting of four contacts contains two odd nodes and, hence, it is Eulerian. Finding an Euler path in this circuit, we tune the module with four contacts (Fig. 4). The considered example shows that the special feature of the proposed topological method is that it, in contrast to the method for synthesizing circuits from functional elements, allows one to realize efficiently not only one formula, but also a system of formulas.

We apply the synthesis method in question to design two-pole contact circuits that realize single Boolean formulas in the considered basis. The simplest two-pole circuit with four odd nodes is shown in Fig. 5. This circuit is impassable, since it contains four odd nodes, and this does not provide 4-universality of the module in the class of arbitrary parallel–serial contact circuits with four contacts. However, this property is fulfilled in the class of *PN*-types of such circuits, since, e.g., instead of the circuit in Fig. 5, we can use a circuit with two odd nodes (Fig. 6), which is Eulerian.

The analysis carried out by the author and described in [13] shows that, for any *PN*-type of Boolean formulas containing at most six letters in the considered basis, we can select a representative that is realized by an Euler circuit that provides the irredundancy of q-universal modules of that type for $q \leq 6$. For $q = 7$, one *PN*-type of Boolean formulas exists, for which a representative that is realizable by an Euler circuit cannot be selected, since all circuits that are functionally equivalent to the circuit in Fig. 7 include four odd nodes.

Realizing this circuit with use of the considered modules is provided due to introducing a redundant (rdn) contact (Fig. 8) analogously to introducing a "bridge" in the Euler problem of the Konigsberg bridges, which gave birth to graph theory [16]. Note that the logical zero is delivered to the winding of the relay corresponding to the "redundant" contact, and this makes this contact always open.

Figure 9 shows the proposed module with eight contacts, which realizes the circuit demonstrated in Fig. 8. Note that an input variable is not delivered to the winding corresponding to the redundant contact. In [13], it is shown that the length of a chain that is required for realizing an arbitrary formula in the basis {&, ∨} containing h letters satisfies the condition

$$h \leq d \leq [(4/3)h - 1/3].$$

Hence, in this case, the redundancy does not exceed 33%, in contrast to the redundancy exceeding 100%, which is obtained by the formula method for modules consisting of elements with unilateral conductance. The approach described in this paper has been used for investigating functionalities of chains consisting of identical resistors. In [17], it is shown, e.g., that a chain consisting of five such resistors, each having the resistance R, can be realized by applying jumpers of 35 different nominal values of resistance, which fall in the range from $R/5$ to $5R$.

CONCLUSIONS

Thus, a fundamentally new class of multifunctional modules consisting of elements with bilateral conductance is proposed; these modules are universal in the class of Boolean formulas in the basis {&, ∨, !}, including h letters which have no element redundancy for $h \leq 6$ due to the equal accessibility of direct and inverse outputs of data sources obtained by tuning at the outputs. It is shown that, for arbitrary values of the number of letters h in the considered Boolean formulas, the element redundancy of such modules cannot exceed 33%. It is significantly smaller than the value of this index for

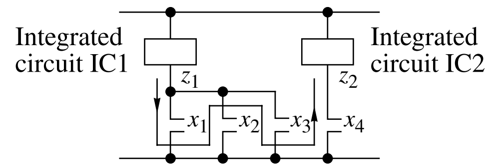

Fig. 3.

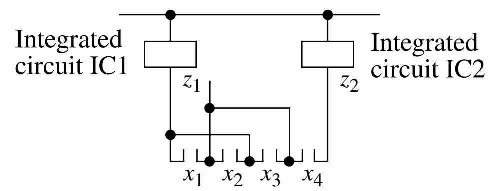

Fig. 4.

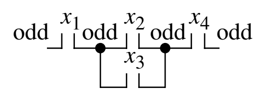

Fig. 5.

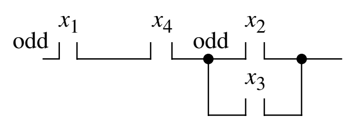

Fig. 6.

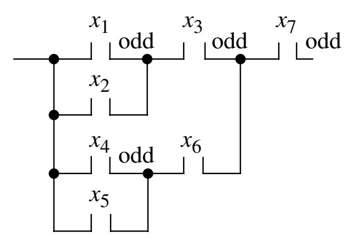

Fig. 7.

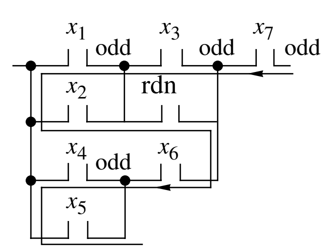

Fig. 8.

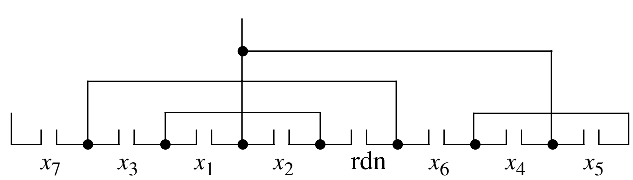

Fig. 9.

known modules consisting of elements with unilateral conductance.

The approach used in designing the proposed modules was applied to the study of the functionalities of homogeneous chains consisting of resistors with intermediate contacts between them.

In particular, such a chain of five resistors generates 35 different nominal values of resistance. A topological method (elaborated on the basis of the Euler graph theory) for realizing multi-output combinational circuits, which makes it possible to apply efficiently the bilateral conductance of elements that form the modules, is presented. Multifunctional modules consisting of elements with bilateral conductance are proposed; these modules are universal under the accessibility of only direct data sources in the class of Boolean formulas in the basis $\{\&, \vee, !\}$ for $h \leq 3$.

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